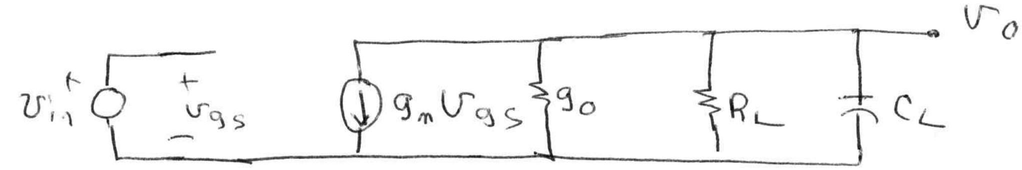
EE 330 Homework 13

Solutions

Problem 1

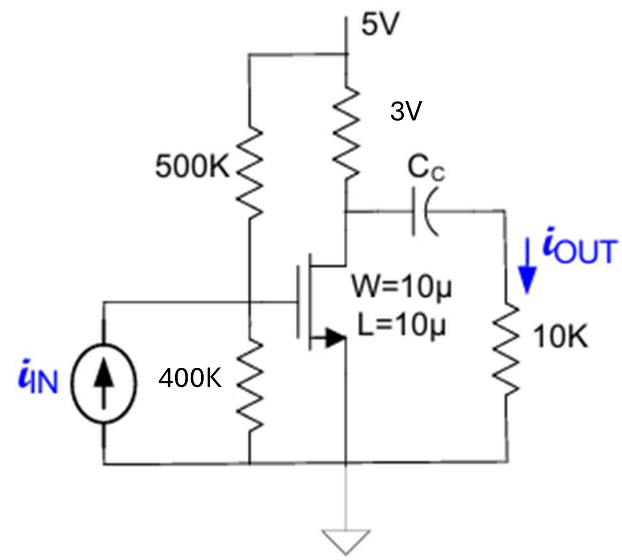


Summing currents on v_o node (with $G_L = 1/R_L$)

$$v_o (sC_L + G_L + g_o) + g_m v_i = 0$$

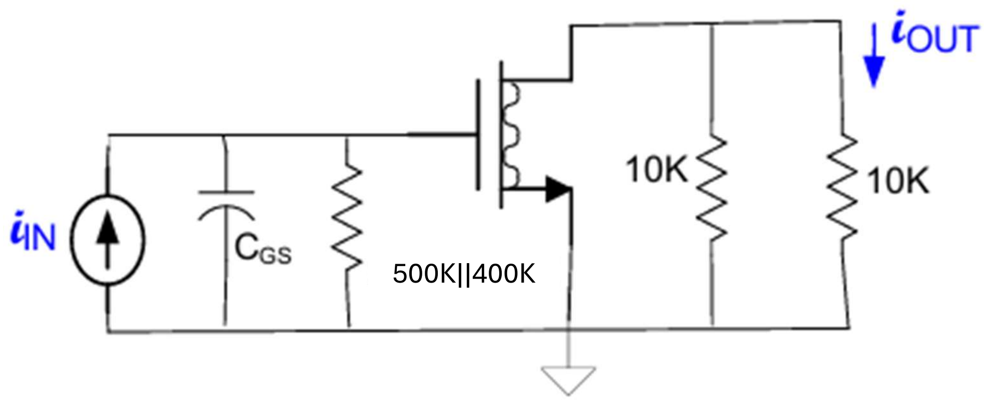
$$\therefore \quad A_v(s) = \frac{v_o}{v_i} = \frac{-g_m}{sC_L + g_o + G_L} \simeq \frac{-g_m}{sC_L + G_L}$$

Problem 2

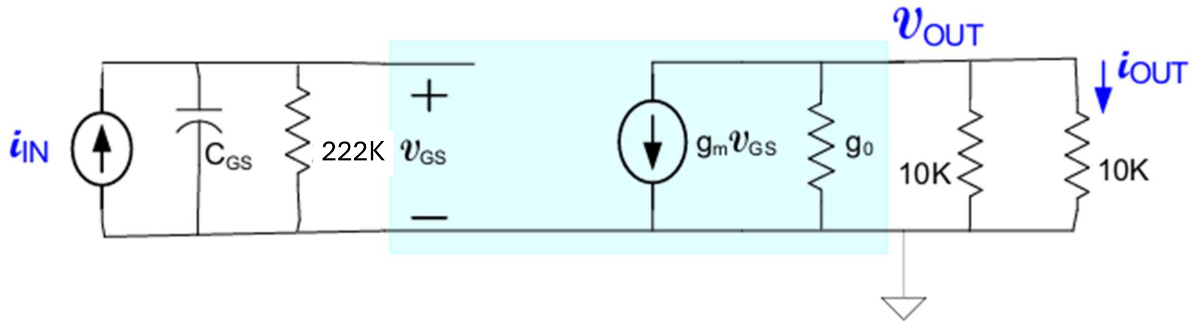


a)

Small signal equivalent circuit including C_{GS} capacitor



Including the MOS small signal model:



b)

Defining R_B=222K, G_B=1/R_B, and G_L=1/R_L=1/10K and summing currents at input and output nodes obtain equations

$$V_{GS}\left(sC_{GS}+G_B\right)=I_{IN}$$

$$V_{OUT}\left(g_o+G_L+G_L\right)+g_m V_{GS}=0$$

$$V_{OUT}G_L=I_{OUT}$$

Eliminating V_{OUT} and V_{GS} from these equations we obtain

$$\frac{I_{OUT}}{I_{IN}}=-\frac{g_m}{R_L\left(g_o+G_L+G_L\right)}\frac{1}{sC_{GS}+G_B}\simeq-\frac{R_B g_m/2}{sR_B C_{GS}+1}$$

c)

Want to obtain

$$\left|\frac{R_B g_m/2}{j\omega R_B C_{GS}+1}\right|=1$$

Which can be written as

$$\frac{\left(R_B g_m/2\right)^2}{1+\left(\omega R_B C_{GS}\right)^2}=1$$

Solving for angular frequency ω we obtain

$$\omega=\frac{\sqrt{\left(R_B g_m/2\right)^2-1}}{R_B C_{GS}}\simeq\frac{g_m}{2C_{GS}}$$

It remains to obtain g_m and C_{GS}. Observe by voltage divider V_{GSQ}=1.33V. So

$$g_m = uC_{ox}\frac{W}{L}\left(V_{GSQ} - V_{TH}\right) = \ 232.5uS$$

$$C_{GS} = C_{OX}WL = 400fF$$

So, unity gain frequency is approximately 290.625M rad/sec or 46.25 MHz

Problem 3 $I_{D2} = \left(\frac{W_2}{W_1}\frac{L_1}{L_2}\right) I_{D1} = (3)(50\mu A) = 150\mu A$

$V_o = 8V - I_{D2} \cdot R = 8V - (150\mu A)(25k) = 4.25V$

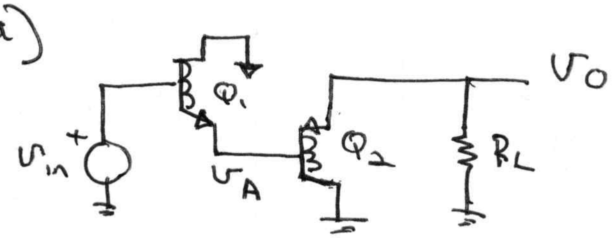
Problem 4 From Lecture Notes $A_v = -\frac{g_{m1}}{2 g_{o1}} \cdot \beta = -\frac{V_{AF}}{2V_t}\beta$

So $A_v \simeq \frac{-200}{50mV} \cdot 100 = -400{,}000$

Problem 5 Define $G_L = \frac{1}{R_L} = \frac{1}{1k\Omega}$

Observe this is a cascade of two CC stages

a)



$A_v = \frac{v_o}{v_A} \cdot \frac{v_A}{v_i}$

$\frac{v_o}{v_A} = \frac{+g_{m2}}{g_{m2}+G_L}$

$R_{in2} \simeq \beta_2 R_L$

$\frac{1}{R_{in2}} = \frac{G_L}{\beta_2}$

$\frac{v_A}{v_{in}} = \frac{+g_{m1}}{g_{m1} + \frac{G_L}{\beta_2}}$

$\therefore A_v = \left(\frac{g_{m2}}{g_{m2}+G_L}\right)\left(\frac{g_{m1}}{g_{m1}+\frac{G_L}{\beta_2}}\right) \simeq 1$

b) $v_{inQ} = 0V$, $V_{OQ} = V_{inQ} - 0.6 + 0.6 = 0V$

c) $R_{in} = r_{\pi 1} + \beta_1 R_{in2} = r_\pi + \beta_1\beta_2 R_L \simeq \beta_1\beta_2 R_L$

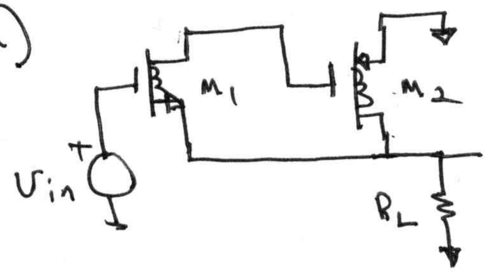
d) If current sources ideal

$v_{OMAX} = V_{CC}$

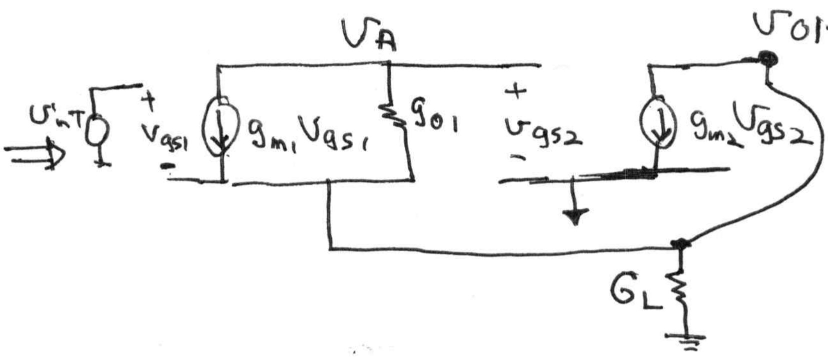
$v_{Omin} = V_{EE} + 0.6V$

Problem 6 Define $G_L = \frac{1}{R_L} = \frac{1}{1K\Omega}$

a)



$\Rightarrow$

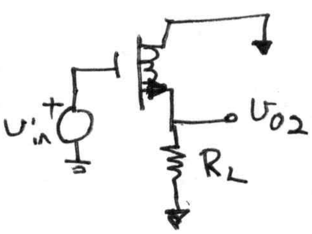


From KCL

$$\left.\begin{aligned} V_{01}(g_{01}+G_L) + g_{m2}V_A &= g_{m1}(V_{in}-V_{01}) + g_{01}V_A \\ V_A &= V_{01} - \frac{g_{m1}}{g_{01}}(V_{in}-V_{01}) \end{aligned}\right\}$$

eliminating V_A, obtain

$$\frac{V_{01}}{V_{in}} = \frac{g_{m1}g_{01} + g_{m1}g_{m2}}{g_{01}[g_{m2}+G_L] + g_{m1}g_{m2}} \simeq \frac{g_{m1}g_{m2}}{g_{m1}g_{m2}}$$



Recognize as common drain amplifier

$\therefore$ $$\frac{V_{02}}{V_{in}} = \frac{g_{m1}}{g_{m1}+G_L}$$

b) $\frac{V_{01}}{V_{in}} \simeq 1$ to find $\frac{V_{02}}{V_{in}}$, need g_{m1}. First obtain I_{DQ}

$$I_{DQ} = \frac{\mu C_{ox} W}{2L}(V_{inQ} - I_{DQ}R - V_{TH})^2$$

solving this equation for I_D, obtain

thus $I_{DQ} = 58.7\,nA \Rightarrow g_m = \sqrt{\mu C_{ox}\frac{W}{L} \cdot 2I_{DQ}} = 383\,\frac{nA}{V}$

$$\frac{V_{02}}{V_{in}} = \frac{383n}{383n + \frac{1}{5K}} = 0.657\ V/V$$

c) Need to reduce $I_{B1} = 5\mu A$, $I_{B2} = 10\mu A$

For circuit on left

$$I_{B2} = \frac{\mu C_{ox} W}{2L}\left(V_{INQ} - V_{OQ} - V_{TH}\right)^2$$

$$10\mu A = 250\mu \left(\frac{10}{4}\right)\left(1V - V_{OQ} - .4\ V\right)^2$$

solving, obtain $V_{OQ} = 0.474\ V$

For circuit on right, found in part b), I_{DQ}

$\therefore\ V_{OQ} = (58.7\mu)(5k) = 0.294\ V$

d) For $V_{INQ} = 4V$, circuit on left has

$V_{OQ} = 3.47\ V$

For circuit on right, must again solve

$$I_{DQ} = \frac{\mu C_{ox} W}{2L}\left(V_{inQ} - I_{DQ}R - V_{TH}\right)^2 \quad \text{for } I_{DQ}$$

with $V_{inQ} = 4V$, obtain $I_{DQ} = 535\ \mu A$

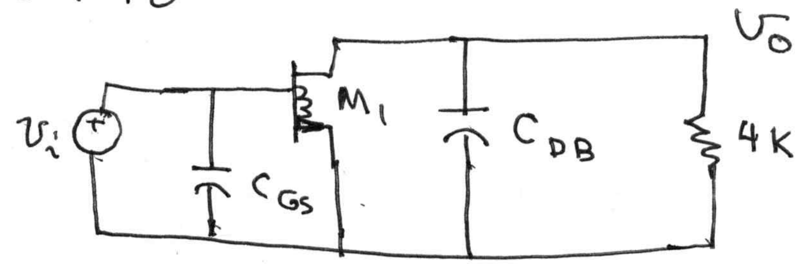
so $V_{OQ} = (I_{DQ})(5k) = 2.67\ V$

Problem 7 From Lecture Slides

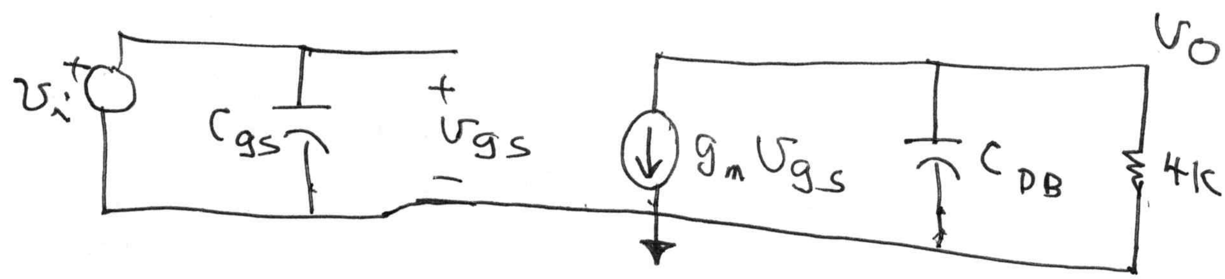
$$A_v \simeq -\frac{g_{m1}}{g_{o1}} = -\frac{\frac{I_{CQ}}{V_t}}{\frac{I_{CQ}}{V_{AF}}} = \frac{V_{AF}}{V_t} = -\frac{100}{25mV} = -4000$$

Problem 8

a)



b)



summing currents at output node with $G_L = \frac{1}{4K}$

$$v_o(sC_{DB} + G_L) + g_m v_i = 0$$

$$\therefore \frac{v_o}{v_{in}} = \frac{-g_m}{sC_{DB} + G_L}$$

c) $\omega_{3dB} = \frac{G_L}{C_{DB}} \Rightarrow f_{3dB} = \frac{1}{(2\pi) R_L C_{DB}}$

$$C_{DB} = C_{SWD} \cdot 52\mu + C_{BOT} \cdot 120\mu^2$$

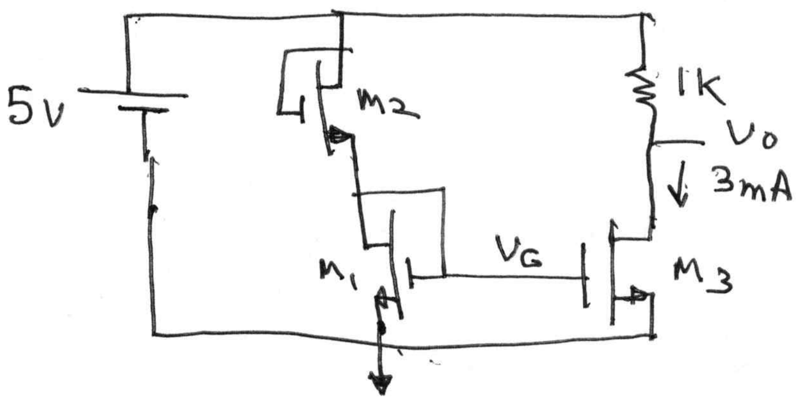
From table $C_{SWD} = 212\ af/\mu$

$C_{BOT} = 942\ af/\mu^2$

$$\therefore C_{DB} = 11.0\ fF + 113\ fF = 124\ fF$$

So $f_{3dB} = \frac{1}{(2\pi)(4K)(124fF)} = 320\ MHz$

Problem 9 One solution



Since $V_o = 2V$, want $V_G < 2V + V_{TH}$ to maintain saturation of M3.

So will set $V_G = 2V$

$$I_{D3} = \frac{\mu C_{ox} W}{2L} (2 - .4)^2$$

$$3E-3 = \frac{250\mu}{2} \cdot \frac{W_3}{L_L} (1.6)^2$$

$\therefore \frac{W_3}{L_3} = 9.38$ Let $L_3 = 1\mu$, $W_3 = 9.38\mu$

Let $\frac{W_1}{L_1} = 9.38$ so unity mirror gain

Let $L_1 = 1\mu$, $W_1 = 9.38\mu$

$\therefore I_{D1} = 3mA$

Consider now M_2 which also has $I_D = 3mA$

$$\therefore 3mA = \frac{\mu C_{ox} W_2}{2L_2} (5 - V_G - V_{TH})^2$$

$$3mA = \frac{250\mu}{2} \frac{W_2}{L_2} (5 - 2 - .4)^2$$

solving

$\frac{W_2}{L_2} = 3.55$ Let $L_2 = 10\mu$, $W_2 = 35.5\mu$